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LH-moment estimation for statistical analysis on the wave crest distributions of a deepwater spar platform model test

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Abstract

The design of fixed and compliant offshore platforms requires the reliable estimation of extreme values with small probabilities of exceedance based on an appropriate probability distribution. The Weibull distribution is commonly utilised for the statistical analysis of wave crests, including near-field wave run-ups. The parameters are estimated empirically from experimental or onsite measurements. In this paper, the data set of wave crests from a Spar model test was statistically analyzed by using the method of LH-moments for parameter estimation of the Weibull distribution. The root-mean-square errors (RMSEs) and the error of LH-kurtosis were used to examine the goodness-of-fit. The results for the first four LH-moments, the estimated parameters, and the probability distributions showed that the level of the LH-moments has a significant influence. At higher levels, the estimation results gave a more focused representation of the upper part of the wave crest distributions, which indicates consistency with the intention of the method of LH-moments. The low tail RMSE values of less than 2.5% demonstrated that a Weibull distribution model estimated by using high-level LH-moments can accurately represent the probability distribution of large extreme wave crests for incident waves, wave run-ups, and moon pool waves. Goodness-of-fit test on the basis of comparison of sampling LH-kurtosis and theoretical LH-kurtosis was recommended as a procedure for selecting an optimum level.

Key words: LH-moments, Weibull distribution, parameter estimation, wave crest, spar platform

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1. Introduction

Determining the extreme values of wave heights, wave run-ups, and wave-induced loads and motions is a central task to the design of offshore structures exposed to ocean environments. Because of the random process of ocean waves, statistical analysis methods and procedures are commonly utilised to derive the appropriate probability distributions and reliably estimate extreme values with small probabilities of exceedance [1]. Recently, there has been much interest in methods of L- and LH-moments for research on reliability and lifetimes in various fields.

As a special case of LH-moments, the method of L-moments was first established by Hosking [2] to characterise the probability distributions and data of extreme events by using the algorithm of probability-weighted moments (PWM) [3]. Although L-moments are analogous to conventional moments, the parameters estimated with the method of L-moments are less biased and more robust for a recorded sample series [4]. Because of its desirable features for estimating extreme events with limited data, the method of L-moments has since become an attractive analytical tool for applied research, and a considerable number of applications have been reported in various research fields, such as regional flood frequency analysis [5]. L-moments have been widely introduced in formulas to estimate the parameters for many probability distributions, including the generalised extreme value (GEV) distribution [6], four-parameter kappa distribution (K4D) [7], generalised logistic distribution (GLD) [8], four-parameter generalised lambda distribution (GLD4) [9], generalised Pareto distribution (GPD) [10], and four-parameter asymmetric exponential power (AEP4) distribution [11].

The method of L-moments has recently been applied to offshore engineering. Izadparast and Niedzwecki [1] used the method of L-moments to derive the parameters of a probability distribution to estimate wave crest distributions. The semi-empirical approach was shown to be effective at representing data for both far-field waves and wave run-ups. Izadparast and Niedzwecki [12] further modified the analytical form of the probability distribution for wave run-ups by introducing an additional empirical parameter with a physical interpretation. All parameters could be estimated directly from the statistics utilising the method of L-moments. The accuracy of the three-parameter distribution model for describing the run-up measurements of a compliant platform was validated through a comparison with the experimental data and other theoretical models. Najafian [13] compared three different methods of moments to estimate the parameters of the Pierson-Holmes distribution [14], which was first introduced as a probability model for Morison wave loads of random waves. In most cases, the sampling variability of the parameter values determined from the two alternative methods of moments was much less than that of the conventional method of moments. Winterstein and MacKenzie [15] improved the four-moment Hermite model by using L-moments rather than conventional moments to estimate the extreme response statistics of nonlinear wind and wave loads on offshore structures.

L-moments are defined as linear combinations of the overall statistics for the whole sample data. However, researchers and engineers are usually interested in the distribution of large values with small probabilities of exceedance rather than the overall statistics. The distribution functions are normally different for reasonable descriptions of a full data series and only the large values. Considering the less-meaningful lower part of distributions tends to obtain insufficient weights for large sample values that actually contain useful information in the upper part of the distributions [16]. Therefore, the method of LH-moments was proposed as a generalisation of L-moments based on linear combinations of higher-order statistics in order to characterise the upper parts of distributions [17]. This method provides an analytical means of fitting a distribution to large sample events without explicit sample censoring.

Since then, LH-moments have been used by several researchers in flood frequency analysis. Hewa et al. [18] applied the method of LH-moments to a GEV distribution for low-flow frequency analysis and revealed that it was more capable of modelling low flows than conventional methods. Meshgi and Khalili [19,20] developed an approach based on LH-moments for GLD and GPD and compared L- and LH-moments for regional flood frequency analysis of the Kharkhe watershed in western Iran. Bhuyan et al. [21] and Deka et al. [22] performed regional flood frequency analysis on the annual maximum rainfall series in India by using LH-moments from the zero level (L0, i.e. L-moments) to the fourth level (L4) to estimate the GEV, GLD, and GPD parameters. Murshed et al. [23] investigated the effect and feasibility of the method of LH-moments for estimating heavy-tail conditions by fitting K4D. They indicated that the method can be useful in many practical applications where even the method of L-moments fails to give a feasible solution.

As a key issue in the design of fixed and compliant offshore platforms, the characteristics of incident wave crests and near-field wave run-ups have been the subject of numerous studies, many of which have focused on the extreme statistics and probability distribution models. As a well-known theoretical model, the Rayleigh distribution is usually utilised for estimating the probability distribution of wave crests based on the assumptions of a narrow-band frequency spectrum and Gaussian distribution for the wave surface elevations. In order to describe nonlinear wave run-up distributions, Kriebel and Dawson [24] proposed another simplified theoretical model that is based on the assumptions that the first- and second-order wave run-ups are phase-locked and that the maxima occur at the same time. The prediction results of this model were shown to be sufficiently accurate through a comparison with more complete second-order numerical simulations for wave run-ups on a vertical cylinder. However, Al-Humoud et al. [25] examined an improved model for ocean surface waves and reported that the qualitative prediction accuracy was elusive. Alternatively, Forristall [26] proposed a two-parameter Weibull distribution model for estimating the probability distributions of wave heights; the parameters were then related to the wave steepness and Ursell parameter to estimate the probability distributions of wave crests [27]. Niedzwecki et al. [28] introduced a model to investigate the wave run-ups due to random seas interacting with compliant platforms by using model test data. They revealed that the wave run-ups on circular columns can be reasonably modelled. Further applications to the wave run-ups on both a single rectangular column and array of similar vertical columns were reported [29]. The improved model was confirmed to be accurate for large wave crests but less for small waves, and the Rayleigh distribution model of the wave crests consistently underestimated the large observed wave run-ups. Tayfun [30] applied the two-parameter Weibull distribution model to estimate the parameters of the quadratic transformation for describing the nonlinear wave crests at transitional water depths. Better accuracy was observed compared with other nonlinear models, but clear deviations were noted for smaller waves. As a generalised distribution widely used in reliability and lifetime studies [31], the three-parameter Weibull distribution has been recommended and confirmed as a good model for the statistical analysis of extreme waves and wave overtopping without the need for data censoring [32,33]. In an analysis of the exceedance probabilities for wave run-ups on a mini-TLP, the parameters of the distribution were proposed to be estimated by using the method of L-moments [1].

For large extreme values with a small probability of exceedance, such as wave crests and wave run-ups, in practice the method of LH-moments tends to be more appropriate than the method of L-moments for the parameter estimation of the probability distributions. Xiao et al. [34] established a methodology based on the method of LH-moments and the three-parameter Weibull distribution model. Generalised formulae for any level of LH-moment were derived from the first four LH-moments and their explicit relationships with the parameters of the Weibull distribution. In this study, the methodology was further applied to analyse the characteristics of incident waves, wave run-ups, and moon pool waves of a deepwater spar platform according to the experimental measurements. The results of the first four LH-moments, estimated parameters, probability distributions, and goodness-of-fit tests for different levels were presented and discussed, including the zero level representing the method of L-moments. Bootstrap analysis was utilized in order to evaluate the uncertainty of extreme predictions of the Weibull models estimated by the method of LH-moments.

2. Methodology

2.1. Definition and direct estimators of LH-moments

The theoretical LH-moments for a real-value random variable X with the quantile function $X(F)$ are defined as the linear combinations of high-order statistics. Wang [17] specifies the first four LH-moments with the levels η = 0, 1, 2… as follows:

$$\lambda_1^\eta = E[X_{(\eta+1):(\eta+1)}] \tag{1}$$

$$\lambda_2^\eta = \frac{1}{2}E[X_{(\eta+2):(\eta+2)} - X_{(\eta+1):(\eta+2)}] \tag{2}$$

$$\lambda_3^\eta = \frac{1}{3}E[X_{(\eta+3):(\eta+3)} - 2X_{(\eta+2):(\eta+3)} + X_{(\eta+1):(\eta+3)}] \tag{3}$$

$$\lambda_4^\eta = \frac{1}{4}E[X_{(\eta+4):(\eta+4)} - 3X_{(\eta+3):(\eta+4)} + 3X_{(\eta+2):(\eta+4)} - X_{(\eta+1):(\eta+4)}] \tag{4}$$

where $E[X_{j:r}]$ denotes the expectation of the jth-order statistic of a sample with size r drawn from the distribution $F(x) = \Pr(X \le x)$. $E[X_{j:r}]$ is given by [2]

$$E[X_{j:r}] = \frac{r!}{(j-1)!(r-j)!}\int_0^1 x(F)F^{j-1}(1-F)^{r-j}\,\mathrm{d}F \tag{5}$$

At the zero level (i.e. η = 0), LH-moments are identical to the L-moments defined by Hostking [2]. For η = 1, 2…, LH-moments are called L1-moments, L2-moments…, respectively. As the level η increases, LH-moments increasingly reflect the characteristics of the upper parts of probability distributions and larger events in data.

Similar to conventional moments and L-moments, the first four LH-moments (i.e. λ_1^η, λ_2^η, λ_3^η, λ_4^η) represent the population measures of the location, scale, skewness, and kurtosis, respectively. According to the features, λ_1^η denotes the location, λ_2^η characterises the spreadness, and λ_3^η and λ_4^η reflect the asymmetry and peak, respectively, of the upper part of a probability distribution. LH-moments can be normalised to define the LH coefficients of the variation, skewness and kurtosis as follows:

$$\tau_2^\eta = \frac{\lambda_2^\eta}{\lambda_1^\eta} \tag{6}$$

$$\tau_3^\eta = \frac{\lambda_3^\eta}{\lambda_2^\eta} \tag{7}$$

$$\tau_4^\eta = \frac{\lambda_4^\eta}{\lambda_2^\eta} \tag{8}$$

where τ_3^η and τ_4^η are analogously to the LH-skewness and LH-kurtosis.

Instead of the standard PWM used in L-moment theory, LH-moments can be estimated with more elegant expressions by using the normalised PWM. This is defined as

$$B_r = \frac{\int_0^1 x(F)F^r \mathrm{d}F}{\int_0^1 F^r \mathrm{d}F} = (r+1)\int_0^1 x(F)F^r \mathrm{d}F = (r+1)\beta_r \tag{9}$$

where β_r is the standard PWM. Wang [17] gives the relationships between the LH-moments and normalised PWMs as follows:

$$\lambda_1^\eta = B_\eta \tag{10}$$

$$\lambda_2^\eta = \frac{1}{2!}(\eta+2)\left[B_{\eta+1} - B_\eta\right] \tag{11}$$

$$\lambda_3^\eta = \frac{1}{3!}(\eta+3)\left[(\eta+4)B_{\eta+2} - 2(\eta+3)B_{\eta+1} + (\eta+2)B_\eta\right] \tag{12}$$

$$\lambda_4^\eta = \frac{1}{4!}(\eta+4)\left[(\eta+6)(\eta+5)B_{\eta+3} - 3(\eta+5)(\eta+4)B_{\eta+2} + 3(\eta+4)(\eta+3)B_{\eta+1} - (\eta+3)(\eta+2)B_\eta\right] \tag{13}$$

These equations can be applied to estimate the LH-moments. However, the estimation is indirect owing to the need to use normalised PWMs. To eliminate this unnecessary complication, Wang [17] developed the following equations as direct unbiased estimators of the first four sample LH-moments at the η-th level instead of using normalised PWMs:

$$\hat{\lambda}_1^\eta = \frac{1}{C_N^{\eta+1}} \sum_{i=1}^{N} C_{i-1}^{\eta} x_{(i)} \tag{14}$$

$$\hat{\lambda}_2^\eta = \frac{1}{2C_N^{\eta+2}} \sum_{i=1}^{N} \left(C_{i-1}^{\eta+1} - C_{i-1}^{\eta} C_{N-i}^{1}\right) x_{(i)} \tag{15}$$

$$\hat{\lambda}_3^{\eta} = \frac{1}{3C_N^{\eta+3}} \sum_{i=1}^{N} \left(C_{i-1}^{\eta+2} - 2C_{i-1}^{\eta+1} C_{N-i}^{1} + C_{i-1}^{\eta} C_{N-i}^{2} \right) x_{(i)} \tag{16}$$

$$\hat{\lambda}_4^{\eta} = \frac{1}{4C_N^{\eta+4}} \sum_{i=1}^{N} \left(C_{i-1}^{\eta+3} - 3C_{i-1}^{\eta+2} C_{N-i}^{1} + 3C_{i-1}^{\eta+1} C_{N-i}^{2} - C_{i-1}^{\eta} C_{N-i}^{3} \right) x_{(i)} \tag{17}$$

where $x_{(i)}, i = 1,2,\cdots, N$ are sample values ranked in ascending order.

$$C_m^j = \binom{m}{j} = \frac{m!}{j!\,(m-j)!} \tag{18}$$

is the binomial coefficient and denotes the number of combinations of any *j* items from *m* items. When $j > m$, the value is equal to zero.

2.2. Weibull distribution and LH-moment estimation

For nonlinear wave crests and run-ups, the probability distribution can be estimated empirically by using the three-parameter Weibull distribution. The probability density function (PDF) $f(x)$, cumulative distribution function (CDF) $F(x)$, and quantile function $x(F)$ of the three-parameter Weibull distribution can be written as follows:

$$f(x) = \frac{\delta}{\kappa} \left(\frac{x-\xi}{\kappa} \right)^{\delta-1} \exp\left(-\left(\frac{x-\xi}{\kappa} \right)^{\delta} \right) \tag{19}$$

$$F(x) = 1 - \exp\left(-\left(\frac{x-\xi}{\kappa} \right)^{\delta} \right) \tag{20}$$

$$x(F) = \xi + \kappa \left[-\ln(1-F) \right]^{1/\delta} \tag{21}$$

where ξ is the location parameter, κ is the scale parameter, and δ is the shape parameter. The distribution of linear wave amplitudes and wave heights is a special case of the Weibull distribution with the parameters $\xi = 0$, $\kappa = \sqrt{2}$, and $\delta = 2.0$. The Weibull distribution is able to capture the nonlinearity in the sample data via the appropriate parameters.

These parameters can be estimated by using different methods; one of these is the utilisation of moments. Specifying the quantile function of the Weibull distribution (i.e. substituting Eq. (21) into Eq. (9)) and applying the binomial theorem yields the expression of the normalised PWMs of the Weibull distribution:

$$B_r = \xi + \kappa\Gamma(1 + 1/\delta)\sum_{j=0}^{r}(-1)^j C_{r+1}^{j+1}(j+1)^{-1/\delta} \tag{22}$$

where $\Gamma(z)$ is the Gamma function.

Further substituting Eq. (22) into Eqs. (10) - (13) yields the first four population LH-moments of the three-parameter Weibull distribution:

$$\lambda_1^\eta = \xi + \kappa\Gamma(1 + 1/\delta)\sum_{j=0}^{\eta}(-1)^j (j+1)^{-1/\delta} C_{\eta+1}^{j+1} \tag{23}$$

$$\lambda_2^\eta = \frac{1}{2!}(\eta+2)\kappa\Gamma(1 + 1/\delta)\sum_{j=0}^{\eta+1}(-1)^j (j+1)^{-1/\delta} C_{\eta+1}^{j} \tag{24}$$

$$\lambda_3^\eta = \frac{1}{3!}(\eta+3)\kappa\Gamma(1 + 1/\delta)\sum_{j=0}^{\eta+2}(-1)^j (j+1)^{-1/\delta} C_{\eta+2}^{j}(j+2) \tag{25}$$

$$\lambda_4^\eta = \frac{1}{4!}(\eta+4)\kappa\Gamma(1 + 1/\delta)\sum_{j=0}^{\eta+3}(-1)^j (j+1)^{-1/\delta} C_{\eta+3}^{j}(j+2)(j+3) \tag{26}$$

These equations denote the relationships between the LH-moments and the parameters of the Weibull distribution. They can then be used for parameter estimation. If $\eta = 0$, the equations give the special case form for the Weibull distribution using L-moments.

2.3. Parameter estimation

Given an ordered sample, the η-th level of the r-th-order sample LH-moments, which is denoted by $\hat{\lambda}_r^\eta$, can be estimated directly by using Eqs. (14) – (17). By equating the population LH-moments λ_r^η in Eqs. (23) – (26) to the corresponding sample LH-moments $\hat{\lambda}_r^\eta$, the unknown parameters of the probability distributions can be calculated sequentially.

For the LH-skewness, the relationship between the sample estimator and population estimator can be derived from Eqs. (15), (16), (24) and (25) to yield

$$\hat{\tau}_3^\eta = \frac{\hat{\lambda}_3^\eta}{\hat{\lambda}_2^\eta} = \frac{(\eta+3)\sum_{j=0}^{\eta+2}(-1)^j (j+1)^{-1/\delta} C_{\eta+2}^{j}(j+2)}{3(\eta+2)\sum_{j=0}^{\eta+1}(-1)^j (j+1)^{-1/\delta} C_{\eta+1}^{j}} \tag{27}$$

Eq. (27) can be used to calculate the unknown shape parameter iteratively. Subsequently, the scale parameter can be calculated with the following equation derived from Eqs. (15) and (24):

$$\kappa = \frac{2\hat{\lambda}_2^{\eta}}{(\eta+2)\Gamma(1+1/\delta)\sum_{j=0}^{\eta+1}(-1)^j (j+1)^{-1/\delta} C_{\eta+1}^{j}} \tag{28}$$

The location parameter is calculated by the following equation derived from Eqs. (14) and (23):

$$\xi = \hat{\lambda}_1^{\eta} - \kappa\Gamma(1+1/\delta)\sum_{j=0}^{\eta}(-1)^j (j+1)^{-1/\delta} C_{\eta+1}^{j+1} \tag{29}$$

Note that the scale parameter can also be calculated by equating the third LH-moments from Eqs. (16) and (25). The result should be the same as that from Eq. (28).

The shape parameter δ cannot be calculated directly by using Eq. (27); an iteration procedure needs to be conducted to obtain an approximate solution. The tolerance of the convergence needs be specified by minimising the following absolute discrepancy:

$$f(\delta) = \left|\tau_3^{\eta} - \hat{\tau}_3^{\eta}\right| \tag{30}$$

The initial value of δ is given by using the formula proposed by Izadparast and Niedzwecki [12]:

$$\delta_0 = \left(1.2362 \times (\hat{\tau}_3^0)^3 + 0.5768 \times (\hat{\tau}_3^0)^2 + 1.8212 \times \hat{\tau}_3^0 + 0.2833\right)^{-1} \tag{31}$$

This is based on L-moment estimation and has an accuracy of better than 2.1×10^{-3} for $0 < \hat{\tau}_3^0 < 0.5$.

By applying the approximate shape parameter δ, the scale parameter κ and location parameter ξ can be directly estimated by using Eqs. (28) and (29).

One of the difficulties of using method of LH-moments in practical applications is the selection of η. In general, one needs to estimate the model parameters for a range of η values and then select the optimum η value through goodness-of-fit tests of probability distributions. The root-mean-square error (RMSE) of the estimated quantiles is commonly used to quantify the tolerance between the measured data and model estimates with the Weibull distribution. Thus, for a random process where N is the size of the sample,

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[x(F_i)_{\mathrm{data}} - x(F_i)_{\mathrm{estimated}}\right]^2} \tag{32}$$

Here, $x(F_i)$ is the quantile function with a cumulative probability of

$$F_i = n_i / (N+1) \tag{33}$$

where n_i is the number of sample values no more than $x_{(i)}$.

As noted earlier, the extreme values of wave crests with a small probability of exceedance are usually the most important concern in practice. Therefore, rather than the overall tolerance using all quantiles, the tail RMSE is evaluated similarly by considering only the large sample values with a probability of exceedance $(1 - F_i) \leq 0.1$.

In addition to the RMSE, another goodness-of-fit test method similar to that proposed by Wang [35] is employed on the basis of comparison of sampling LH-kurtosis estimates and theoretical LH-kurtosis values. The relative discrepancy between the theoretical LH-kurtosis τ_4^η of the fitted distribution and the sampling LH-kurtosis $\hat{\tau}_4^\eta$ is normally expressed as a percentage value as

$$\mathrm{ERR}_{\mathrm{kur}} = \frac{\left|\tau_4^\eta - \hat{\tau}_4^\eta\right|}{\hat{\tau}_4^\eta} \times 100\% \tag{34}$$

3. Experimental data

To evaluate the statistics and probability distributions of wave crests based on the method of LH-moments, experimental data were obtained from a model study investigating the hydrodynamic response of a truss spar platform. The experiment was carried out at the deepwater offshore basin of Shanghai Jiao Tong University, China. The basin is 50 m in length, 40 m in width, and 10 m in maximum effective water depth. A large-area movable floor allows the flexible modelling of the water depth from 0 m to 10 m as required. A secondary movable floor in the deep pit with a diameter of 5 m further allows the modelling of the water depth from 10 m to 40 m. Various environments can be modelled, including collinear and non-collinear waves, currents and winds simulated by using two multi-flap wave generators, a deepwater global current generation system, and axial wind fan matrix.

The truss spar model consisted of a square soft tank, truss section, hard tank with helical strakes, and the topside. The linear scale ratio between the prototype and model was 1:60. Table 1 lists the main parameters of the spar. The freeboard of the Spar hull is 15.28 m. Triple helical strakes with an offset of 120° were installed to suppress vortex-induced motions. The height and pitch were set to 10% and three times the hard tank diameter, respectively. Fig. 1 shows the model of the truss spar in the basin. The natural periods of heave, roll, and pitch motions of the free floating Spar were measured as 22.5 s, 52.2 s, and 53.0 s, respectively. The natural period of piston mode inside the moon pool was calculated as 16.9 s.

The design water depth of the spar platform was 1500 m. The spread mooring system of the truss spar consisted of 12 semi-taut chain-wire-chain mooring lines, which were grouped in three bundles 120°

apart. Each group had four mooring lines, and the interval angle was 5°. Based on the scale ratio of 1:60 and the maximum water depth of the wave basin, a truncated mooring system was designed and modelled for a truncated water depth of 600 m. In total, seven models of top-tensioned risers, including six production risers and one drilling riser, were vertically installed through the moon pool of the spar and in the deep pit without truncation. Additional details including the mooring and riser systems are given in [36].

A number of ocean environments were considered in the experiment; the typical operational and survival conditions were selected for the present data analysis and are given in Table 2. Here, H_s is the significant wave height, σ_ζ is the standard deviation, T_p is the peak wave period, V_w is the mean wind speed, and V_c is the mean surface current speed. A steady wind and uniform current were simulated. The typical JONSWAP wave spectrum was applied in the simulation of irregular waves with a peakedness factor of γ = 3.3. The time duration of each irregular wave realisation was approximately 23.2 min at the model scale, which corresponded to 3 h at the full scale.

In the experiment, the data measurements included the six degrees-of-freedom (DOF) motions, top tensions of all mooring lines and risers, translational accelerations, and relative wave elevations. For the data analysis, the incident waves, wave run-ups, and relative wave elevations in the moon pool were considered. These were measured with a number of resistance-type wave probes. Fig. 2 shows a plan view of the tested spar model and the wave probe locations. Wave probes 1 and 3 were installed close to the outside surface of the hard tank to measure the upstream and lateral wave run-ups, respectively. Wave probe 2 was installed at the center of the moon pool. The three wave probes were located under the cellar deck and thus the height limitation in the run-up measurements will be theoretically the still-water air gap value between the deck bottom and the water surface, which is 22.86 m in prototype and 0.381 m in model scale. In order to validate the LH-moment estimations, the relative wave elevations measured by wave probes 1 and 2 were analysed.

Prior to the placement of any model in the basin, the specified irregular waves were calibrated such that the resulting power spectral densities closely matched the corresponding target spectra. Fig. 3 shows the calibration results; the good agreement indicates that high-quality wave generation was achieved. The time series of the calibrated waves were further used in the statistical analysis as incident waves corresponding to the operational and survival cases.

4. Data analysis and results

4.1. Basic statistics

Basic statistical analysis was first performed on the measured time series of the wave surface elevations. Table 3 summarises the results. Here, σ is the standard deviation, N_c is the number of wave crests, and T_z is the mean zero up-crossing period. The standard deviations of the incident waves agreed well with the values specified in Table 2. By comparing the basic statistics of the wave run-ups and moon pool waves to those of the incident waves, the effects of the incident waves passing over the spar hull could be observed. The increased standard deviations and wave cycles along with the shortening of the mean zero up-crossing periods indicate that the wave field around the spar hull was amplified as a consequence of the interactions between the incident, diffracted, and radiated waves. On the other hand, the decreased standard deviations and wave cycles along with the lengthening of the mean zero up-crossing periods indicate that the wave field in the moon pool was suppressed as a consequence of the shielding effect by the spar hull. The response spectra of the wave run-ups and moon pool wave elevations presented in Fig. 4 further revealed the amplifying and shielding effects by comparing to the incident wave spectra shown in Fig. 3.

4.2. Estimated LH-moments

From the basic statistical analysis of the time series, samples of wave crest amplitudes were obtained and ranked in an ascending order for further analysis with the method of LH-moments. The mean value of the measured wave surface elevations was excluded from these derived amplitudes.

In order to evaluate the dependence of the estimation results of important characteristics, such as LH-moments, Weibull parameters, fitted probability distributions, tail RMSEs and errors of LH-kurtosis, on the level of LH-moments, a set of values for η from 0 to 30 with an interval of 1 was adopted for the analysis. As discussed in the reference [12], the physical characteristics of large crests in nonlinear fields are different from the characteristics of small crests and this requires different sets of parameters for small and large crest heights. This indicates that the need for using very high η values arises because the Weibull distribution chosen for modeling the overall probability of wave crests may be far from appropriate for describing the extreme wave crests. Nevertheless, the level could not be set too high because the binominal coefficient represented by Eq. (18) grows exponentially with it, which tends to result in numerical instability.

Fig. 5 shows the variation in the first four LH-moments of the incident waves, wave run-ups, and moon pool waves under operational and survival conditions and normalised by using the standard deviation σ_ζ of the incident waves. The LH-moments clearly showed a significant dependence on the level η. All LH-moments varied at different levels and exhibited a significant difference compared to the L-moments at $\eta = 0$.

The LH-location increased monotonically as the level increased. This indicates that the estimation was more focused on the upper part of the wave crests. This variation is consistent with the intention of the method of LH-moments. In comparison with those under the operational condition, the normalised LH-location values under the survival condition were slightly smaller for the wave run-ups, while larger values were clearly observed for the moon pool waves. This indicates a smaller reduction of waves in the moon pool at higher sea states. In regard to the incident waves, the normalised LH-location values were slightly larger at low levels and smaller at high levels under the survival condition.

When the level was increased from 0 to 30, the LH-scale values initially decreased rapidly and then stabilised at high levels. This indicates that the variation in the wave crests tended to decrease as more upper part data were considered. A similar trend was observed for the fourth-order LH-moments. As shown in Fig. 5, the third-order LH-moments clearly indicated the opposite trend except for the incident wave under the survival condition, although it tended to similarly be uniform at high levels.

As shown in Fig. 5, the values of the second-, third-, and fourth-order LH-moments for the wave run-ups were larger under the survival condition and smaller under the operational condition compared to those for the incident waves. This indicates that the wave-hull interactions in benign sea states during the operational condition play a positive role on wave run-ups due to the compliant characteristic of the spar platform. For high sea states during the survival condition, the wave-hull interactions tend to significantly enhance the wave run-ups outside the spar hull, which results in more nonlinearity. For the moon pool waves, the values of the LH-moments were significantly smaller than those of the incident waves. This was expected owing to the shielding effect of the spar hull and was similar to the basic statistics.

4.3. Parameter estimation using LH-moments

In terms of the calculated sample LH-moments, three parameters of the corresponding Weibull distribution can be derived by using Eqs. (27) - (29). An iterative scheme was developed to solve the approximated shape parameter δ, and the tolerance of convergence was specified as 10^{-4} in the present study. The consequent discrepancy $f(\delta)$ was less than the specified tolerance.

Fig. 6 presents the estimated shape, scale, and location parameters respect to different levels of LH-moments for the incident waves, wave run-ups, and moon pool waves under the operational and survival conditions. The location and scale parameters were normalised by using the standard deviation σ_ζ of the incident waves.

All three parameters varied considerably for different levels of LH-moments, which clearly indicates the sensitivity of the estimated parameters at the level η. The Rayleigh model is often used because of its simple form with the shape parameter δ = 2 and location parameter ξ = 0; Fig. 6 shows that the shape and location parameters were generally close to these two values for L-moments, i.e. η = 0. When the level η was increased, the shape and location parameters varied significantly compared to the values at η = 0, which resulted in clearly different probability distributions for the upper part of the wave crests. However, the extreme predictions based on the quantile function (see Eq. (21)) of the Weibull distribution depend on all three parameters, i.e. (ξ, κ, δ). For a certain small probability of exceedance, the extreme quantile $x(F)$ increases with increasing κ while decreases with increasing δ. As shown in Fig. 6, the variations of κ and δ are almost identical whereas the variation of ξ is opposite. This indicates that there is cancellation effect of the parameters on the extreme predictions. The variation of the parameters was larger under the survival condition than under the operational condition, which indicates that large wave crests exhibit more nonlinearities in the incident waves, wave run-ups, and moon pool waves.

4.4. Comparison of Weibull distributions

By substituting the estimated parameters (i.e. δ, κ, and ξ) into Eq. (20), the cumulative probability function $F(x)$ and exceedance probability function $1-F(x)$ at different levels of LH-moments can be obtained. On the other hand, by applying Eq. (33) to the wave crest amplitudes obtained from the basic statistical analysis, the cumulative and exceedance probabilities corresponding to the recorded quantiles can be calculated directly as the measured data for comparison with the results estimated by using

LH-moments. From the total levels of 0 – 30, four typical levels were selected for the comparison with the measured data: η = 0 (representing L-moments), 5, 10, and 30. Fig. 7 compares the exceedance probability distributions for the incident waves, wave run-ups, and moon pool waves under the operational and survival conditions.

As discussed in previous sections, the estimated probability distributions tend to increasingly represent the upper part of the extreme values as the level increases. Fig. 7 clearly shows this trend. The probability distributions based on L-moments (i.e. η = 0) agreed with the measured data very well at the lower part of wave crests but were clearly worse at the upper part. As the level increased, the estimated probability distributions more closely matched the upper part of the measured wave data while simultaneously exhibiting a greater discrepancy with the lower part.

Because the upper distribution corresponding to large quantiles with a specified small probability of exceedance are typically of more interest, the high-level LH-moments tend to be more reliable for probability analysis than L-moments. As shown in Fig. 7, the large quantiles estimated with L-moments were clearly more or less than those estimated with high-level LH-moments. This indicates that using L-moments tends to cause an overestimation or underestimation of the large extreme values. Nevertheless, the tail distributions presented in Fig. 7 do not show large variations with η ranging from 0 to 30 because of the aforementioned cancellation effect of Weibull parameters which may change significantly. The difference between the estimated results at various high levels was not significant, which indicates that the level does not need to be too high to decrease the possibility of numerical instability.

Using high-level LH-moments provides an analytical means of reliably fitting a distribution in the upper tail without explicit sample censoring as adopted in the peaks-over-threshold (POT) method. The POT method considers only the extreme quantiles over a threshold value for modeling the tail of a distribution. Similar to that adopted in the GPD for extreme value forecasting [37], the POT method can be incorporated to the L-moments estimation to improve the performance of Weilbull distribution in the upper tail. As an example, a sensitivity study was conducted on the data sample of the operational incident wave by selecting a range of threshold values (corresponding exceedance rates are λ = 0.8 ~ 0.2) and the fitted Weibull distributions were compared to the experimental data in Fig. 8. It can be clearly observed that the fitted Weibull distributions with exceedance rates of $\lambda \leq 0.4$ resulted in good representation of the upper tail, similar to that using high-level LH-moments at η = 10 in Fig. 7(a). The tail RMSEs normalised by H_s can be reduced from 4.8% to 1.2% when the exceedance rate was selected to be $\lambda \leq 0.4$, similar to that using LH-moments at $\eta \geq 6$.

In regard to the incident waves, it can be observed in Figs. 7(a) and 7(b) that the peaks of measured wave crest heights are $5.3\sigma_\zeta$ and $4.4\sigma_\zeta$ under operational and survival conditions, respectively. The tail distribution estimated by L-moments tends to underestimate the extreme values in operational condition while overestimate those in survival condition. The operational incident wave shows more significant level of non-linearity than the survival incident wave. This may be related to the capability of the wave maker in the offshore basin. The wave height under survival condition is closer to the capability and thus more limitation exists in generating extreme wave crests. Moreover, the generated waves under survival condition suffer more from wave breaking because of higher steepness and this may result in reduction of extreme wave crests.

The probability distributions of the wave run-ups and moon pool waves were larger under survival conditions than under operational conditions, which indicates that more serious wave-hull interactions result in a higher wave crest response outside the spar hull and inside the moon pool. Under survival condition, the upper limitation of the wave run-up measurements is normalized as 6.1 and the freeboard of the Spar hull is normalized as 4.07. As observed in Fig. 7(d), the extreme measurements on the tail of wave run-ups are lower than the limitation but higher than the freeboard. In this case, the waves pass over the top of the Spar hull and green water occurs. This phenomenon may contribute to the difference of the physical characteristics of the wave run-up measurements. In addition, wave breaking may occur and thus influence the extreme measurements. For the moon pool waves, the extreme values at small probabilities of exceedance were significantly higher under survival conditions than under operational conditions, although they were still much lower than those of the incident waves and outside wave run-ups. This indicates that the spar hull provides a good shielding effect.

4.5. Goodness-of-fit evaluation

Because they represent the tolerance between the estimated quantiles and the corresponding measured data, the RMSE and tail RMSE were used to examine the overall goodness-of-fit to the data and the accuracy of the estimation with the upper part of the distributions. The tolerance was normalised by using the significant wave height $H_s = 4\sigma_\zeta$. This allowed the error estimation to be interpreted as a percentage of the error relative to the significant wave height of the incident wave.

Fig. 9 presents the estimated RMSE and tail RMSE at different levels of LH-moments for the incident waves, wave run-ups, and moon pool waves under operational and survival conditions. The RMSE and tail RMSE values clearly showed significant increases and decreases, respectively, as the level increased. The RMSE values were less than 2% for L-moments at η = 0 but reached up to 15% at higher levels of LH-moments. Simultaneously, the tail RMSE values were close to 5% for L-moments at η = 0 and decreased rapidly to uniform values of less than 2.5% after a level of about η = 10. The low tail RMSE value indicates that the Weibull distribution model estimated by using high-level LH-moments can accurately represent the probability distributions of extreme values of the wave crests for not only incident waves but also the wave run-ups and moon pool waves.

Fig. 9 also shows that the RMSE and tail RMSE values were clearly higher under survival conditions than under operational conditions except for the incident waves. This agrees with the comparison of exceedance probability distributions in Fig. 7, which showed clear discrepancies for the largest quantiles of the wave run-ups and moon pool waves. The complex wave-hull interactions under survival conditions tend to result in possibly abnormal wave run-ups with extremely small probabilities of exceedance and increase the difficulty of estimating the probability distributions properly with the normal Weibull distribution model.

The relative errors between theoretical and sampling LH-kurtosis values are shown in Fig. 10. Unlike the trend of RMSE, the error of the LH-kurtosis fluctuates significantly with the η level of LH-moments. An optimum level can be determined in accordance with the minimum error, which means the fitted Weibull distribution has the best representation of the statistics of the data sample. For evaluating goodness-of-fit by using L-moments (η=0) and LH-moments, the relative errors of LH-kurtosis and the tail RMSEs at zero level and the optimum level are listed in Table 4. In comparison with the results at η=0, the errors of LH-kurtosis at the optimum level reduced significantly to sufficiently small values less than 2.2%, except that the optimum level for the incident wave in survival condition is equal to zero and the error is close to zero. Compared to the tail RMSE, the error of LH-kurtosis is more sensitive to the level and thus is more suitable to be the criteria for selecting an optimum level. The optimum η can be up to 20 and even 30, as shown in Table 4, which illustrates the need of a wide range of η for selection. Therefore, goodness-of-fit tests on the basis of comparison of sampling LH-kurtosis estimates and theoretical LH-kurtosis values of the fitted Weibull distributions are recommended as a procedure for selecting an optimum η.

4.6. Uncertainty of extreme predictions

The main purpose of the fitted probability distributions using parameter estimation methods is to estimate reliable extreme statistics with small probabilities of exceedance. As a powerful computer-intensive method typically used to determine the bias and variance associated with sample estimates of a parameter of interest [12], the bootstrap analysis is utilized in order to evaluate the uncertainty of extreme predictions of the Weibull models estimated by the method of LH-moments. The variance and RMSE of the extreme quantiles with probabilities of exceedance of 0.1, 0.05, 0.01, 0.005, and 0.001 are estimated using 1000 bootstrap samples. Table 5 presents the RMSEs of extreme quantiles with different probabilities estimated by L-moment models at η=0 and LH-moment models at the optimum levels. Here the RMSE values are normalized to percentage errors by comparing to the quantiles estimated using the original data sample. It can be seen that the RMSE increases monotonously with the increase of probability of exceedance and thus the sampling variability is larger in the tail fitting. Moreover, the RMSE values obtained by LH-moments at the optimum level are slightly higher than those by L-moments at η=0. However, even though the level η becomes very high (e.g. η=20 and 30 for wave run-ups under operation and survival conditions, respectively), the RMSE values with a very low probability of exceedance of 0.001 are still less than 10% (e.g. 5.1% and 6.2% for wave run-ups). In fact, the RMSE values presented in Table 5 are less than 7% for all studied cases. This indicates that both L-moment and LH-moment models are less sensitive to the uncertainty of data samples and are confirmed to be less biased and more robust for a recorded sample series.

5. Conclusions

The estimation method of LH-moments for the three-parameter Weibull distribution was applied to the data set of wave crests from a Spar model test for statistical analysis of incident waves, wave run-ups, and moon pool waves. The statistics and spectra of the wave run-ups and moon pool waves in comparison with those of incident waves revealed the amplifying and shielding effects of the Spar hull. The influence of the level of LH-moments tended to be significant; clear variations were seen in the results for the LH-moments, estimated parameters, and probability distributions at different levels. When the level was increased, the estimation results presented a more focused representation of the upper part of the wave crests, which is consistent with the intention of the method of LH-moments. As the level was increased from $\eta = 0$ (representing L-moments), the estimated values initially varied rapidly and then stabilised at high levels, which makes sense intuitively. The level does not need to be too high to avoid numerical instability.

The RMSE and the error of LH-kurtosis were used to examine the goodness-of-fit. The RMSE and tail RMSE values showed significant increases and decreases, respectively, as the level increased. A similar result was observed for the comparison of exceedance probability distributions. Because the upper distribution of wave crests is typically of more interest, the low tail RMSE values of less than 2.5% indicates that the Weibull distribution model estimated with high-level LH-moments can accurately represent the probability distributions of extreme wave crests for incident waves, wave run-ups, and moon pool waves. Goodness-of-fit test results showed that the error of LH-kurtosis is more sensitive to the level than the tail RMSE and thus is more suitable to be the criteria for selecting an optimum level. Bootstrap analysis indicated that both L-moment and LH-moment models are less sensitive to the uncertainty of data samples.

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